EELE 250: Circuits, Devices, and Motors

Lecture 11

Assignment Reminder

- Read 4.1 - 4.3 AND 5.1 - 5.4
- Practice Problems:
  - P3.60, 3.64, 3.72
  - P4.3, 4.8, 4.9, 4.23, 4.37, 4.38

- D2L Quiz #5 by 11AM on Monday 30 Sept.

- Lab #4 will be performed next—be sure to do the pre-lab assignment calculations!

Sinusoidal Current and Voltage

- $v(t) = V_m \cos(\omega t + \theta)$

- $\omega = 2 \pi f$ [radians / sec]
- f = frequency [cycles / sec] or [Hz]
- $T = 1 / f$ = period [sec]

- Root mean square (RMS) concept

Sinusoids

- Which is the correct relationship between sine and cosine?
    A. $\cos(\theta) = \sin(\theta + \pi/2)$
    B. $\cos(\theta) = \sin(\theta - \pi/2)$
    C. $\cos(\theta) = \sin(\theta + \pi)$
    D. $\cos(\theta) = \sin(\theta - \pi)$
    E. $\cos(\theta) = -\sin(\theta)$

Sinusoids

- Which is the correct relationship between sine and cosine?
  A. $\sin(\theta) = -\cos(\theta)$
  B. $\sin(\theta) = \cos(\theta + \pi/2)$
  C. $\sin(\theta) = \cos(\theta - \pi)$
  D. $\sin(\theta) = \cos(\theta - \pi/2)$
  E. $\sin(\theta) = \cos(\theta + \pi)$

Phasors

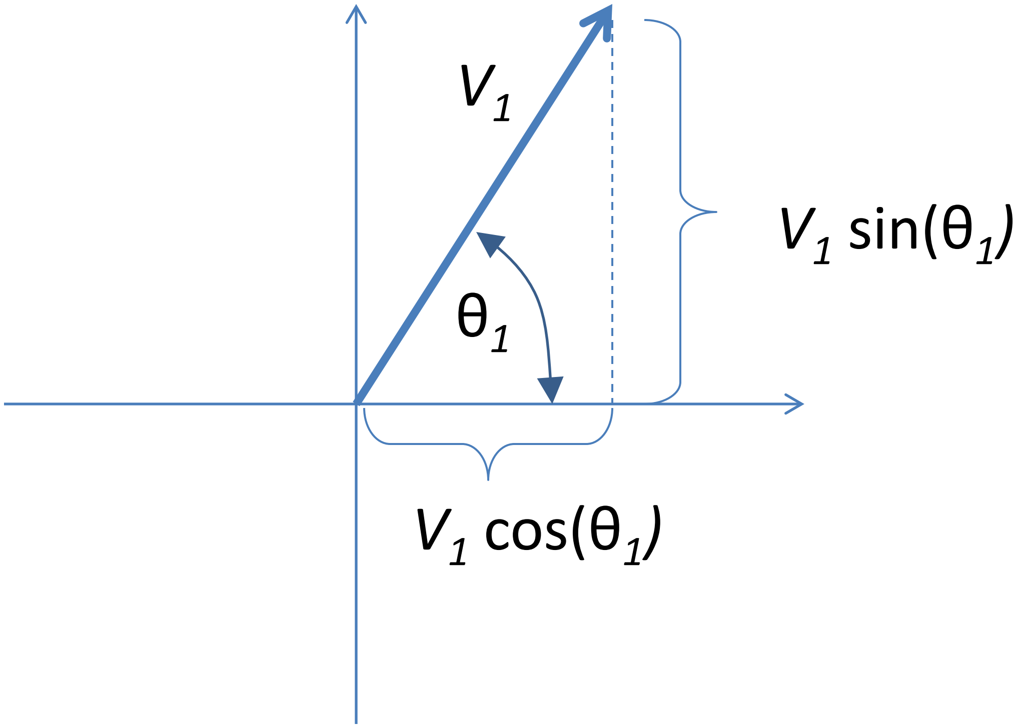
- Represent a sinusoid $v(t) = V_1 \cos(\omega t + \theta_1)$ as a vector of length V_1 and angle θ_1 with respect to the real axis
- This vector is equivalent to a *complex number*

  real part is $V_1 \cos(\theta_1)$

  and

  imaginary part is $V_1 \sin(\theta_1)$
- (Polar form vs. rectangular form)

Phasors (cont.)



Phasors (cont.)

- Circuits with sinusoidal signals often result in KVL or KCL expressions like:

$$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3)$$

It is a pain to add these signals via trigonometric identities!

Fortunately, it is easier to add using phasors: add the vectors as complex numbers.

Phasors (cont.)

$V_1 \cos(\omega t + \theta_1) + V_2 \cos(\omega t + \theta_2) + V_3 \cos(\omega t + \theta_3)$

Phasors: $V_1 \angle\theta_1 + V_2 \angle\theta_2 + V_3 \angle\theta_3$

Real parts: $V_1 \cos(\theta_1) + V_2 \cos(\theta_2) + V_3 \cos(\theta_3)$

Imag parts: $V_1 \sin(\theta_1) + V_2 \sin(\theta_2) + V_3 \sin(\theta_3)$

Sum phasor:

sqrt(real^2 + imag^2) $\angle$atan(imag/real)

Complex impedances

- Inductor: $v(t) = \mathrm{L}\ di/dt$
- If $i(t) = I_m \cos(\omega t)$, then $v(t) = -\omega I_m \mathrm{L} \sin(\omega t)$

  ⇨ *note that* $-\sin(\omega t) = \cos(\omega t + 90^\circ)$
- As phasors:

  $\boldsymbol{I} = I_m \angle 0^\circ$ $\boldsymbol{V} = \omega I_m \mathrm{L} \angle 90^\circ$

which means:

$$\boldsymbol{V} = (\omega \mathrm{L} \angle 90^\circ) \cdot (\boldsymbol{I})$$

Note: $\omega \mathrm{L} \angle 90^\circ$ is the complex number $j \omega \mathrm{L}$

Complex Impedances (cont.)

- $\boldsymbol{V} = (\ \omega\ \mathrm{L} \angle 90^\circ\ )\cdot(\boldsymbol{I}) = (j\ \omega\ \mathrm{L}\ )\cdot(\boldsymbol{I})$
- Ohm's Law: V = I·R, can be generalized to

  $\boldsymbol{V} = \boldsymbol{I}\cdot \boldsymbol{Z}$, where ***Z*** is the *impedance*.
- ***Z*** can be a real or a complex number
  - Impedance of a resistor: **Z** = R
  - Impedance of an inductor: **Z** = j ω L
  - Impedance of a capacitor: **Z** = 1/(j ω C)

Complex Impedances (cont.)

- NOTE that the impedance of an inductor or capacitor depends upon the sinusoidal frequency, *ω*

$$Z_L = j\omega L \qquad\qquad Z_C = \frac{1}{j\omega C}$$

- Impedance magnitude of inductor goes *up* as frequency increases
- Impedance magnitude of capacitor goes *down* as frequency increases

Summary and Review

- Represent a group of sinusoids with the same frequency as *phasors*
- Add phasors by interpreting them as complex numbers
- Generalize Ohm's Law to be **V** = **I Z**
- Impedance of a resistor: **Z** = R
- Impedance of an inductor: **Z** = j ω L
- Impedance of a capacitor: **Z** = 1/(j ω C)